Chapter 6
State Estimation

Part 4
6.4 Satellite Navigation Systems
6.4 Satellite Navigation Systems
– 6.4.1 Introduction
– 6.4.2 Implementation
– 6.4.3 State Measurement
– 6.4.4 Performance
– 6.4.5 Modes of Operation
– Summary
Outline

• 6.4 Satellite Navigation Systems
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  – 6.4.2 Implementation
  – 6.4.3 State Measurement
  – 6.4.4 Performance
  – 6.4.5 Modes of Operation
  – Summary
Introduction

• Satellite navigation has been called the next utility.
  – Like electric power and telephony.
  – Works continuously everywhere 24 hours a day.

• Revolutionized shipping, surveying, geophysics, all resource industries.

• Sales of GPS equipment and services are
  – expected to grow to $21.5 billion in 2008
  – up from $13 billion in 2003

• Many industries now completely dependent on it. (repercussions like power grid blackout).
History

• Marine Radio Navigation now obsolete
  – Military TRANSIT system in use since 1964
  – Civilian STARFIX system in use since 1986

• Satellite Radio Navigation
  – US Global Positioning System (developed by US DOD)
  – Soviet GLONASS (virtually identical to GPS)
  – European Galileo system launching since 2005
Availability

- Visible above mask angle of ~ 10 degrees.
- **Always at least 4 visible.**
- Excellent coverage of poles (but see GDOP).
- **GPS requires line of sight:**
  - Can’t be used underground.
  - Can’t be used underwater.
  - Can’t be used in thick forest.
  - Can’t be used around too many tall buildings.
Outline

• 6.4 Satellite Navigation Systems
  – 6.4.1 Introduction
  – 6.4.2 Implementation
  – 6.4.3 State Measurement
  – 6.4.4 Performance
  – 6.4.5 Modes of Operation
  – Summary
6.4.2.1 Satellites and Ground Stations

• 27 (24 plus 3 extra) earth satellites and in six orbits.
  – Actually 31 in orbit now
• Orbits inclined at 55 degrees to equatorial plane.
• Circular orbits 11,000 miles in amplitude
• Repeat exactly twice per sidereal day (12 hour orbits)
6.4.2.1 Satellites and Ground Stations

- Five ground stations spaced in longitude around the globe
- One is designated Master Control Station (MCS):
  - Tracks satellite positions very precisely.
  - Maintains overall system time standard.
- Satellites are updated on their position and clock bias for later retransmission to receivers
6.4.2.2 Signals

• Like most radio, the signals are modulated carrier signals.

• **Two carriers are used**, designated L1 (1575.42 MHz), and L2 (1227.60 MHz).
  – This allows measurement of atmospheric delay

• **Modulators include many things:**
  – C/A (coarse acquisition) PRN (pseudorandom noise) codes
  – P (precise) PRN code
  – secret Y code
  – Navigation message
  – Each satellite has its own distinct codes.

• **Nav Message includes:**
  – Handover word (system time of week) to aid in P code acquisition.
  – Accurate ephemeris for the satellite.
  – Less accurate ephemeris for all other satellites.

<table>
<thead>
<tr>
<th></th>
<th>C/A</th>
<th>P</th>
<th>NAV</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>L2</td>
<td></td>
<td>*</td>
<td>x</td>
</tr>
</tbody>
</table>

* P is encrypted on L2
And known as P(Y)
6.4.2.3 Receiver Operation

- Receiver duplicates PRN codes internally to match with received signals.
- Correlation peaks at correct delay.
- PRN codes allow use of small antennae and handheld receivers are available.
- Receiver is a matched filter.
- Some receivers function solely as precision time or frequency standards.
Outline

• 6.4 Satellite Navigation Systems
  – 6.4.1 Introduction
  – 6.4.2 Implementation
  – 6.4.3 State Measurement
  – 6.4.4 Performance
  – 6.4.5 Modes of Operation
  – Summary
6.4.3.1 Position Measurement

(Principle of Operation)

• Basic idea is **range triangulation**.
• Constellation of satellites in earth orbit.
• Receivers pick up satellite radio transmissions on the ground.
• **Satellites broadcast** signals. This has military advantages:
  – Receivers do not answer - stealthy.
  – No limit on number of users.
• This also means **anyone can use it**.
  – Civilian use is part of the federal plan.
  – **Usage is free** once you buy the equipment.
6.4.3.1 Position Measurement
(Position Measurement)

- Receivers measure range to satellites.
- Satellites transmit their positions, called “ephemeris data”. 
  \[ r_1 = \sqrt{(x - x_1)^2 + (y - y_1)^2} \]
  \[ r_2 = \sqrt{(x - x_2)^2 + (y - y_2)^2} \]
- 3D case is based on intersections of spheres.
- Ambiguity is resolved in several ways.
- Range is measured as time of transit of radio signal times wave speed.
- Sensitivity is about 1 meter in 50 million!
6.4.3.2 Time Measurement

- Satellites use atomic clocks
  - synchronized with the ground stations to the nearest nanosecond or so.
- Receivers use cheap crystal oscillator clocks.
  - Hence the receiver clocks are always off.
  - Use one extra satellite to measure this user clock bias.
- Clock bias causes range measurement errors so range measurements are called pseudoranges.

All agree at one value of clock bias.
4D Situation

• Since (GPS) satellites are synchronized, clock errors cause identical equivalent range errors for all satellites.

\[
\begin{align*}
    r_1 &= \sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 + c \Delta t} \\
    r_2 &= \sqrt{(x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 + c \Delta t} \\
    r_3 &= \sqrt{(x - x_3)^2 + (y - y_3)^2 + (z - z_3)^2 + c \Delta t} \\
    r_4 &= \sqrt{(x - x_4)^2 + (y - y_4)^2 + (z - z_4)^2 + c \Delta t}
\end{align*}
\]

• Solve these for x, y, z, \( \Delta t \) ! No big deal.
6.4.3.3 Wave Speed Prediction and Measurement

• Deviation from the nominal ‘c’ is significant enough to matter greatly.

• Two methods are used:
  – Prediction: Mathematical models of atmospheric delay (satellites broadcast model coefficients).
  – Measurement: Differential delay measurement for two carrier frequencies (delay is frequency dependent).
6.4.3.4 Velocity Measurement

• Principle is the Doppler frequency shift.
• Frequency shift is proportional to “range rate” caused by:
  – Velocity of satellite on orbit.
  – Velocity of earth’s surface caused by earth’s spin.
  – Velocity of vehicle on the earth’s surface.
• Differentiate equation A to see how to convert four range rate observations into geocentric Cartesian velocity.
6.4.3.5 Orientation Measurement

• Principle is measurement of differential positions of several antennae fixed to a rigid vehicle.
• Measure differential carrier phase - “codeless” operation
• Each of three satellites provides a projection of the baseline vector (between the two antennae) onto each satellite beam axis.
• Solve for $\Delta r$ vector in world frame.
• Need second baseline to determine rotation around this $\Delta r$.

\[
\Delta \phi_1 = \Delta r \cos \theta_1 = (\Delta \mathbf{r} \cdot \hat{r}_1) / |\hat{r}_1| \\
\Delta \phi_2 = \Delta r \cos \theta_2 = (\Delta \mathbf{r} \cdot \hat{r}_2) / |\hat{r}_2| \\
\Delta \phi_3 = \Delta r \cos \theta_3 = (\Delta \mathbf{r} \cdot \hat{r}_3) / |\hat{r}_3| \\
\begin{bmatrix}
\hat{r}_{1x} & \hat{r}_{1y} & \hat{r}_{1z} \\
\hat{r}_{2x} & \hat{r}_{2y} & \hat{r}_{2z} \\
\hat{r}_{3x} & \hat{r}_{3y} & \hat{r}_{3z}
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix} =
\begin{bmatrix}
\Delta \phi_1 \\
\Delta \phi_2 \\
\Delta \phi_3
\end{bmatrix}
\]
6.4.3.6 Geodetic Coordinate Systems

- GPS uses the WGS-84 Earth-Centered Earth-Fixed (ECEF) System.
- Centered at Earth’s mass center.
  - Easy to express satellite orbits.
- Earth polar radius is 21 Km less than equatorial.
  - Latitude is defined as shown opposite.
Outline

• 6.4 Satellite Navigation Systems
  – 6.4.1 Introduction
  – 6.4.2 Implementation
  – 6.4.3 State Measurement
  – 6.4.4 Performance
  – 6.4.5 Modes of Operation
  – Summary
6.4.4.1 Sources of Error

- Pseudorange error sources add in rms sense.
- Then, multiply result by GDOP.

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Nominal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmospheric Delays</td>
<td>4 meters</td>
</tr>
<tr>
<td>Satellite Clock &amp; Ephemeris</td>
<td>3 meters</td>
</tr>
<tr>
<td>Multipath</td>
<td>1 meters</td>
</tr>
<tr>
<td>Receiver electronics and vehicle dynamics</td>
<td>1 meters</td>
</tr>
<tr>
<td>Total</td>
<td>5.2 meters (nominal)</td>
</tr>
</tbody>
</table>
Selective Availability

- Turned off in 2000.
- SPS error was reduced from 100m to 20 m.

- Deliberate accuracy degradation by the DOD based on two techniques:
  - Adding “noise” to the ephemeris data.
  - Dithering the satellite clock.
- S/A is very long in period so it cannot be filtered in real time. It is robust to most ideas you may have for eliminating it - except differential techniques.
- Pseudorange error for stationary receiver might look as shown above.
6.4.4.1.1 Atmospheric ("Group") Delay

- Worth up to **30 meters range error** if you do nothing.
  - Varies with time and position.
  - Different for each satellite at any time.
- Two kinds, ionospheric & tropospheric.
- **Ionospheric**
  - caused by charged particles causing diffraction (bending).
  - Varies by factor of 5 from day to night.
  - Varies by factor of 3 due to elevation angle.
  - Affected by solar magnetic activity.
  - Greatest at poles and equator.
  - Cannot be modeled adequately.
- **Tropospheric**
  - caused by water content changing index of refraction
  - 2.3 meters at zenith.
  - 25 meters at horizon.
  - Can be easily modeled.
6.4.4.1.2 Multipath

- Additional signal arrives through reflected non-line-of-sight path which is then out of phase with the direct signal.
- Result is destructive interference
- Substantial above water (water is an L band radio mirror).
- Usually worth less than 1 meter range error.
- Pronounced when receiver is close to surface.
- Moral is to mount antennae as high as possible.

\[ \Delta L = c\Delta t \approx 2h \sin \theta \]
6.4.4.1.3 Geometric Dilution of Precision

- Usually from 4 to 6
- As high as 20.
- Very poor at poles (all satellites at horizon)
- In the context of GPS, five terms are defined:
  - TDOP - time dilution of precision (range equivalent)
  - PDOP - position dilution of precision (3D)
  - HDOP - horizontal dilution of precision
  - VDOP - vertical dilution of precision
  - GDOP - geometric dilution of precision
- Related by:
  \[ \text{GDOP} = \sqrt{(\text{PDOP})^2 + (\text{TDOP})^2} \]
  \[ \text{PDOP} = \sqrt{(\text{HDOP})^2 + (\text{VDOP})^2} \]
6.4.4.2 Measures of “Accuracy”

• **Circular Error Probable (CEP)** (related to Spherical Error Probable, Probable Error) is NOT the most probable error.
  
  – 50% of measurements have errors above, 50% below.

• **2Drms** is twice the standard deviation. 95% of measurements should be less than 2drms.
  
  \[ 2 \times \text{drms} \approx 2.5 \times \text{CEP} \]

• Vendors quote anywhere from 1 mm to 1/10 kilometer accuracy. Need to look deeper to understand why.
Outline

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  – 6.4.5 Modes of Operation
  – Summary
6.4.5 Modes of Operation

• Tradeoff exists between accuracy, frequency of operation, and excursion over which it applies.

• All modes of operation trade one of these against the others.
  – Coded modes – use the PRN codes
  – Codeless modes – use the carrier directly
6.4.5.1 Coded and Codeless Modes
(Coded Modes)

• Coded Modes
  – Highest update rate is a few Hz.
  – Relies on the PRN codes.
  – Accuracy depends on which code you use:
    – C/A code is 300 meter pulse wavelength
    – P code is 30 meter pulse wavelength

• Absolute
  – Adopt geocentric coordinate system.
  – Accuracies about 10 meters (called Standard Positioning Service or SPS).

• Relative
  – This is the error in the difference of two measurements taken by the same receiver separated in space and time.
  – Can only remove position and time independent bias.
  – Relative accuracy degrades to SPS in only a few minutes.

• Repetitive
  – This is the difference in two measurements taken at the same place at different times.
  – Can only remove time independent bias.
6.4.5.2 Code Phase Differential GPS

- Use two receivers, near each other, simultaneously with a communication link between them.
  - Relies on redundant communications channel.
  - Creates another opportunity for signal occlusion.
- Relies on reference “all in view” set called the base station.
- Pseudorange error is known at the reference site since it does not move (and its position is surveyed or considered the origin).
- Transmit this to the moving set.
- 3-5 meter accuracy possible up to 20 Km away.
6.4.5.2 Code Phase Differential GPS

• Basic comms topology:
3.4.5.4 GPS Augmentation (WAAS)

- Wide Area Augmentation System
  - Available in Western Hemisphere.
  - Europe, India, Japan have their own

- Geostationary satellites used to boost the effective range of dedicated base stations.

- WAAS Satellites send corrections for 3 most significant error sources:
  - clock
  - ephemeris
  - atmosphere corrections
STARFIRE

- Clock and ephemeris corrections distributed by satellite – like WAAS

- Global coverage based on 25 ground stations around the world.
- Expensive dual-channel receiver reads Y code on L1 and L2 frequencies
- Computes Ionosphere delay
  - Does not decrypt the military Y code
  - Just measures phase difference
- Accuracies / Repeatabilities
  - SF1 service (1 $\sigma$) $\rightarrow$ 1 m / 15 cm
  - SF2 service (1 $\sigma$) $\rightarrow$ 4.5 cm / 2.5 cm
Codeless Differential Modes

- Techniques used in surveying. Available everywhere.
- All are “differential” modes.
- Based on carrier phase measurements.
- Accuracies of few millimeters plus 2 ppm excursion possible.
- Static Surveying
  - Most reliable and most accurate method.
  - One receiver in known position.
  - Other one in unknown position.
  - Need 1 to 3 hours dwell per point.
  - Answer available only after postprocessing.
  - No lock on carrier required.
  - 5 mm accuracy achievable.
- Kinematic Surveying
  - Occupy point for fraction of a second.
  - Continuous lock required on 4 satellites.
  - This is now real time for robotics applications. 1cm accuracy typical when signal is strong.
Overall Accuracy

• A Note about Accuracy Measurements
  – Accuracy depends on confidence measure -- CEP (SEP) (50%), 2 drms (95%), 3drms (99%), etc.
  – Varies with location, even time of day (GDOP, atmospheric delays)

• Numbers below are for 2 drms, i.e. 95% confidence measure
Standard Positioning System (SPS)

- 3 meter horizontal accuracy (95% confidence)
- 5 meter vertical accuracy (95% confidence)
- 167 nanosecond time accuracy

![Graph showing GPS SPS position accuracy as a function of latitude.](image)

2drms (2 sigma)
All-in-view
1 sec averaging
24 hour test
Outline

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  – 6.4.3 State Measurement
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  – 6.4.5 Modes of Operation
  – Summary
Summary

• GPS provides continuous (2Hz) radio-triangulated position everywhere on the planet and in the near space region where line of sight to the sky is available.

• Receivers measure range to the satellites and time is considered a 4th unknown.

• Civilians are presently denied access to the more precise absolute navigation signals and therefore get an order of magnitude reduction of performance.
  – However, use of differential techniques makes this irrelevant in most applications.
Summary

• There are many signals available and many processing techniques which lead to a universe of performance specs with vary based on:
  – use of code correlation, code phase, or carrier phase
  – which code (C/A, P, Y), if any, is used
  – which carrier (L1, L2, Both) is used
  – availability of differential corrections
  – frequency of updates
  – extent of excursion
  – dynamics of host vehicle